# MAXIMUM TEMPERATURE OF A FIRE SHEET (SCREEN) WITH TUBES WELDED ON THE OUTSIDE 

A. P. Baskakov and E. V. Il'ina

UDC 620.9:662.92; 658.264


#### Abstract

A formula for calculating the temperature distribution over the width of a fire sheet with water-cooled tubes welded on the outside with boundary conditions of the second and third kinds has been obtained. The heat flux from incandescent combustion products is incident on the interior surface of the sheet. A strong influence on the overheating of the sheet at the center of the gap between the tubes is exerted by the step between by tubes, the heat flux incident on the sheet, the thickness and width of the weld, and the thermal conductivity of the metal. The thermal resistance of the heat transfer from the tube wall to boiling water is low; therefore, it is of little importance.


Gases escaping from converters and other copper-smelting furnaces have a temperature of $1200-1450^{\circ} \mathrm{C}$ and a high dust content. It is not practical to cool them in common waste-heat boilers because of the rapid covering of the boilers with dust. This investigation is part of a complex development of the "Uralénergotsvetmet" Public Corporation on creation of boilers suitable for cooling of such gases with the aim of obtaining a vapor and preparing a gas for sulfuric-acid production. Thus, the escaping gases containing sulfurous anhydride are recovered after cooling and are not released into the atmosphere.

The chemical composition of converter gases is as follows: $\mathrm{N}_{2}=81.15 \%, \mathrm{SO}_{2}=11.35 \%, \mathrm{H}_{2} \mathrm{O}=5 \%, \mathrm{CO}_{2}=$ $2 \%$, and $\mathrm{O}_{2}=0.5 \%$. The radiation of the flux is mainly determined by the radiation of dust and ash particles, but, since the composition of the dust is unknown, the radiation is evaluated quite roughly. The spectral characteristics are $k_{\mathrm{b} . \text { att }} \cong 0.03(\mathrm{~m} \cdot \mathrm{MPa})^{-1}$ and $a_{\mathrm{fl}} \cong 0.47$. For the given composition of the gases and a concentration of the ash in them of $c_{\text {ash }} \approx 0.02 \mathrm{~kg} / \mathrm{m}^{3}$ the enthalpy of the gases in the temperature interval $1200-1450^{\circ} \mathrm{C}$ is $1685-2040 \mathrm{~kJ} / \mathrm{m}^{3}$ [1]. At a temperature of $1200-1450^{\circ} \mathrm{C}$, the average coefficient of thermal conductivity of the gases is $\lambda_{\mathrm{g}}=0.186 \mathrm{~W} /(\mathrm{m} \cdot \mathrm{K})$ and the average coefficient of kinematic viscosity is $v_{\mathrm{g}}=377.8 \cdot 10^{-6} \mathrm{~m}^{2} / \mathrm{sec}$ [1]. The average velocity of the gases in the gas duct is $w_{\mathrm{g}} \cong 10 \mathrm{~m} / \mathrm{sec}$.

Screens cooling the gases are made in the form of steel fire sheets with tubes which are welded to them on the outside and in which water or a water-vapor mixture is circulating [2]. In this work, we propose an approximate method of calculation of the temperature distribution in such a structure (Fig. 1). It is assumed that the tubes are welded to a sheet by a weld of constant thickness $\delta_{\text {weld }}$ along their entire length (see Fig. 1). The temperature of boiling water in the tubes is constant along their length; hence, the metal temperature changes only in the plane of the drawing (Fig. 1). The thermal-conductivity coefficients $\lambda$ of the metal of the tubes, the fire sheet, and the weld are considered to be equal and independent of temperature.

When the heat from the hot gases to a fire sheet is transferred mainly by radiation, the density of the heat flux incident on the sheet is taken to be constant (boundary conditions of the second kind).

Let us consider a portion of width $S$ equal to half the distance between the tube axes and of length equal to unity (for example, 1 m ). We subdivide the entire structure into three separate elements (hatched in different directions on the left in Fig. 1b): 1) portion of the fire sheet of width $L$ considered as a rod whose temperature changes only with length and is constant over the thickness in each cross section; 2) portion of width $F$ of the fire sheet, the weld, and the tube through which, first, the heat received on this portion by the fire sheet from the gases (the "path" of this heat flux is equal to $\left(\delta_{\text {st }}+\delta_{\text {weld }}\right)$ ), second, the heat supplied along the sheet (its "path" on this portion is equal to $\left(\delta_{\text {weld }}+0.5 \delta_{\mathrm{st}}\right)$, and, third, the heat received by the sheet on a portion beyond the reach of welding is transferred by

Ural State Technical University, Ekaterinburg, Russia; email; pte@mail.ustu.ru. Translated from InzhenernoFizicheskii Zhurnal, Vol. 75, No. 6, pp. 185-190, November-December, 2002. Original article submitted January 28, 2002; revision submitted May 21, 2002.


Fig. 1. Actual diagram of welding of the tubes to the fire sheet (a) and the equivalent design diagram (b).
heat conduction to the interior surface; the surface of portion 2 giving up heat to the boiling water by convection has a temperature $t_{\mathrm{w}} ; 3$ ) tube wall considered as an infinite rod cooled by boiling water and having a temperature of $t_{\mathrm{w}}$ at the base.

The heat-balance equation for portion 2 has the form

$$
\begin{equation*}
Q_{\mathrm{st}}+q\left(F+\delta_{\mathrm{unw}}\right)=\alpha\left(t_{\mathrm{w}}-t_{0}\right) F+Q_{\mathrm{t}}, \tag{1}
\end{equation*}
$$

where $Q_{\mathrm{t}}$ is the heat flux arriving at the tube wall from portion 2 and transferred then to the cooling liquid beyond the portion $F$.

To determine the value of $Q_{\mathrm{t}}$ we will consider the tube wall as a thin rod whose one end has a temperature $t_{\mathrm{w}}$. The heat exchange between the rod and the cooling liquid with a temperature $t_{0}$ occurs from the lateral surface of the rod and is prescribed by the boundary conditions of the third kind [3]:

$$
q_{\mathrm{t}}=\alpha\left(t-t_{0}\right) .
$$

Since the wall thickness is much smaller than half the perimeter of the tube, we take a one-dimensional temperature distribution in the rod. In view of the high intensity of the heat transfer to the boiling liquid, the rod can be considered to be infinite. If the coefficient of heat transfer $\alpha$ from the wall to the cooling liquid remains constant along the perimeter of the tube (for example, in cooling the tube by not boiling running water), the temperature along the rod changes exponentially and

$$
\begin{equation*}
Q_{\mathrm{t}}=\vartheta_{1} \sqrt{\alpha \lambda u f}, \tag{2}
\end{equation*}
$$



Fig. 2. Toward derivation of the formula for the temperature distribution over the length of the fire sheet.
where $\vartheta_{1}=t_{\mathrm{w}}-t_{0}, u=1$, and $f=\delta_{\mathrm{t}}$.
In developed nucleate boiling of water, the heat-transfer coefficient depends on the heat-flux density $q_{\mathrm{t}}$ and the saturation pressure. In the range of pressures $(1-200) \cdot 10^{5} \mathrm{~Pa}$, we can employ M . A. Mikheev's design equation [4]

$$
\begin{equation*}
\alpha=3.4 q_{\mathrm{t}}^{2 / 3} \cdot\left(10^{-5} p\right)^{0.18} /\left(1-4.5 \cdot 10^{-8} p\right) . \tag{3}
\end{equation*}
$$

In this case, the temperature distribution along the rod will be different. Substituting $q_{\mathrm{t}}=\alpha\left(t-t_{0}\right)$, where $t$ is the temperature of the tube wall at a given site, into formula (3), upon transformations we obtain

$$
\alpha=\beta\left(t-t_{0}\right)^{2},
$$

here $\beta=\left[3.4 \cdot\left(10^{-5} p\right)^{0.18} /\left(1-4.5 \cdot 10^{-8} p\right)\right]^{3}$.
Considering the tube wall as a rod giving up heat on one side, we obtain the equation of change of the temperature along the perimeter of the tube:

$$
d^{2} \vartheta / d x^{2}=a \vartheta^{3},
$$

where $a=\beta u / \lambda f$ and $\vartheta=t-t_{0}$.
Having introduced the variable $P=d \vartheta / d x\left(d^{2} \vartheta / d x^{2}=P(d P / d \vartheta)\right)$, we obtain the equation

$$
P(d P / d \vartheta)=a \vartheta^{3}
$$

whose integral is

$$
P^{2}=0.5 a \vartheta^{4}+c_{1},
$$

where $c_{1}$ is the integration constant. It is found from the condition $P=d \vartheta / d x=0$ for $\vartheta=0$, i.e., $t=t_{0}$, whence $c_{1}=0$.

It is unnecessary to seek the temperature distribution along the perimeter of the tube ("rod"). It will suffice to just determine the derivative at the base of the "rod" (at $x=0$, at the site of welding), since the quantity of heat transferred over the tube wall from the zone of welding is

$$
Q_{\mathrm{t}}=-\left.\lambda f(d \vartheta / d x)\right|_{x=0}
$$

Then

$$
\left.(d \vartheta / d x)\right|_{x=0}=P= \pm \sqrt{0.5 a}\left(t_{\mathrm{w}}-t_{0}\right)^{2}, \quad Q_{\mathrm{t}}=\sqrt{0.5 u \lambda f}\left[3.4 \cdot\left(10^{-5} p\right)^{0.18} /\left(1-0.0045 \cdot 10^{-8} p\right)\right]^{3}\left(t_{\mathrm{w}}-t_{0}\right)^{2}
$$

Substitution of the last expression for $Q_{\mathrm{t}}$ into Eq. (1) will not lead to a substantial refinement of the result in view of the smallness of the influence of this term but it will make the solution more cumbersome because of its nonlinearity. Therefore, in this work, we employ the "classical" solution (2), which is simpler for analysis and is based on the condition $\alpha=$ const.

Taking into account the low value of $Q_{\mathrm{st}}$ as compared to $\alpha\left(t_{\mathrm{w}}-t_{0}\right) F$, we take $q_{\mathrm{t}}=q$ in formula (3) to simplify the solution.

To find the heat flux along the steel sheet $Q_{\text {st }}$ we will consider it as a rod with a cross-sectional area of $\delta_{\text {st }} \cdot 1$ $=\delta_{\mathrm{st}}$ (Fig. 2).

The heat-balance equation for the element $d x$ of the surface of the sheet of unit length is

$$
\begin{equation*}
q d x \cdot 1=d Q_{\mathrm{st}}=d q_{\mathrm{st}} \delta_{\mathrm{st}} \cdot 1 \tag{4}
\end{equation*}
$$

According to the Fourier law, we have

$$
q_{\mathrm{st}}=-\lambda d t / d x
$$

whence

$$
d q_{\mathrm{st}}=-\lambda\left(d^{2} t / d x^{2}\right) d x
$$

Having substituted $d q_{\mathrm{w}}$ into Eq. (4), we obtain the differential equation

$$
\begin{equation*}
d^{2} t / d x^{2}=-q /\left(\lambda \delta_{\mathrm{st}}\right) \tag{5}
\end{equation*}
$$

whose general solution has the form

$$
\begin{equation*}
t=-q x^{2} /\left(2 \lambda \delta_{\mathrm{st}}\right)+c_{1} x+c_{2} \tag{6}
\end{equation*}
$$

We find the values of the constants $c_{1}$ and $c_{2}$ from the boundary conditions at $x=L: Q_{\text {st }}=0$, i.e., $d t / d x=$ $-q L /\left(\lambda \delta_{\mathrm{st}}\right)+c_{1}=0$ and at $x=0$ (at the boundary of the weld): $t=t_{\text {cent }}$, whence it follows that $c_{1}=q L /\left(\lambda \delta_{\mathrm{st}}\right)$ and $c_{2}=t_{\text {cent }}$.

Thus, we have

$$
\begin{equation*}
t-t_{\text {cent }}=q L^{2}\left[\frac{x}{L}-0.5(x / L)^{2}\right] /\left(\lambda \delta_{\mathrm{st}}\right) \tag{7}
\end{equation*}
$$

The sheet temperature is maximum at $x=L$. Equation (7) yields

$$
\begin{equation*}
t_{\max }-t_{\mathrm{cent}}=0.5 q L^{2} /\left(\lambda \delta_{\mathrm{st}}\right) \tag{8}
\end{equation*}
$$

The heat flux $Q_{\text {st }}$ approaching the weld (cross section $x=0$ ) along the sheet is equal, naturally, to

$$
\begin{equation*}
\left.Q_{\mathrm{st}}\right|_{x=0}=q L \tag{9}
\end{equation*}
$$

Substituting the quantities found into Eq. (1)

$$
q L+q\left(F+\delta_{\mathrm{unw}}\right)=\alpha\left(t_{\mathrm{w}}-t_{0}\right) F+\left(t_{\mathrm{w}}-t_{0}\right) \sqrt{\alpha \lambda \delta_{\mathrm{t}}}
$$

we obtain the expression for the temperature of the interior tube surface in the zone of welding:

$$
\begin{equation*}
t_{\mathrm{w}}=t_{0}+q S /\left(\alpha F+\sqrt{\alpha \lambda \delta_{\mathrm{t}}}\right) \tag{10}
\end{equation*}
$$

We believe that the heat $Q_{\mathrm{st}}=q L$ supplied along the fire sheet to the zone of welding propagates in a onedimensional manner in this zone from the center of the sheet thickness of temperature $t_{\text {cent }}$ to the interior surface of the tube, overcoming the thermal resistance of half the sheet thickness, the weld thickness, and the tube wall. The heat $q\left(F+\delta_{\text {unw }}\right)$ received by the screen in the welding zone is added to it. Then we have

$$
\begin{equation*}
q L+q\left(F+\delta_{\mathrm{unw}}\right)=\lambda F\left(t_{\mathrm{cent}}-t_{\mathrm{w}}\right) /\left(0.5 \delta_{\mathrm{st}}+\delta_{\mathrm{weld}}\right) \tag{11}
\end{equation*}
$$

whence the temperature at the center of the sheet $t_{\text {cent }}$ in the welding zone is

$$
\begin{equation*}
t_{\text {cent }}=t_{\mathrm{w}}+q S\left(0.5 \delta_{\mathrm{st}}+\delta_{\mathrm{weld}}\right) / \lambda F \tag{12}
\end{equation*}
$$

Having substituted the found quantity $t_{\text {cent }}$ into (8) with account for expression (10) for $t_{\mathrm{w}}$, we will finally have the formula for the maximum temperature of the fire-sheet surface written in dimensionless form:

$$
\begin{equation*}
\frac{\left(t_{\max }-t_{0}\right) \lambda}{q S}=\frac{1}{(\alpha F / \lambda)+\sqrt{\alpha \delta_{\mathrm{t}} \lambda}}+\frac{0.5 \delta_{\mathrm{st}}+\delta_{\mathrm{weld}}}{F}+\frac{0.5 L^{2}}{S \delta_{\mathrm{st}}} \tag{13}
\end{equation*}
$$

For specific calculations we take the pressure $p=4.5 \cdot 10^{5} \mathrm{~Pa}$, which is common with waste-heat boilers and corresponds to a saturation temperature of $t_{0}=256^{\circ} \mathrm{C}$. We employ the following numerical values characteristic of waste-heat boilers with screens in the form of fire sheets: $S=40.5 \cdot 10^{-3} \mathrm{~m}, \delta_{\mathrm{t}}=5 \cdot 10^{-3} \mathrm{~m}, \delta_{\mathrm{st}}=8 \cdot 10^{-3} \mathrm{~m}$, and $\delta_{\text {weld }}$ $=8 \cdot 10^{-3} \mathrm{~m}$. The thermal-conductivity coefficient for 12 Kh 1 MF steel of which the screen walls are made is 40 $\mathrm{W} /(\mathrm{m} \cdot \mathrm{K})$ in the temperature range $330-350^{\circ} \mathrm{C}$ [5].

Knowing the height of the weld $\delta_{\text {weld }}=8 \mathrm{~mm}$, the inside diameter of the tube $d_{\mathrm{t}}=42 \mathrm{~mm}$, the minimum height of the air gap between the tube and the fire sheet $\Delta=2 \mathrm{~mm}$, and the width of the unwelded portion of the sheet $\delta_{\text {unw }}=5 \mathrm{~mm}$, we can easily find the distance $F$.

According to the theorem on the intersection of chords, we have

$$
\left(F+\delta_{\text {unw }}\right)\left(F+\delta_{\text {unw }}\right)=\left[d_{\mathrm{t}}-\left(\delta_{\text {weld }}-\Delta\right)\right]\left(\delta_{\text {weld }}-\Delta\right),
$$

whence

$$
F=\sqrt{\left[\left(d_{\mathrm{t}}-\left(\delta_{\mathrm{weld}}-\Delta\right)\right)\left(\delta_{\mathrm{weld}}-\Delta\right)\right]}-\delta_{\mathrm{unw}}=10 \mathrm{~mm}
$$

From the data of the "Uralénergotsvetmet" Public Corporation, in the zone of the highest temperatures of the gases, the heat flux to the screen varies in the range of $35-210 \mathrm{~kW} / \mathrm{m}^{2}$. Substituting these values into formula (3), we find $\alpha=9000-30,000 \mathrm{~W} /\left(\mathrm{m}^{2} \cdot \mathrm{~K}\right)$.

After the substitution of all the values into formula (13) we obtain $t_{\max }=345-747^{\circ} \mathrm{C}$.
The temperature $t_{\min }$ of the lower surface of the sheet in the region of the weld is easy to calculate from the formula

$$
\begin{equation*}
t_{\min }=t_{\mathrm{cent}}+0.5 \delta_{\mathrm{st}} q / \lambda \tag{14}
\end{equation*}
$$

In the example in question, $t_{\min }=313-555^{\circ} \mathrm{C}$.
For the heat-transfer coefficient $\alpha=9000 \mathrm{~W} /\left(\mathrm{m}^{2} \cdot \mathrm{~K}\right)$ (the minimum for our example), the first term on the right-hand side of formula (13) is equal to $1 /(2.250+1.061)=0.302$, while for the maximum coefficient $\alpha=30,000$ $\mathrm{W} /\left(\mathrm{m}^{2} \cdot \mathrm{~K}\right)$ it is $1 /(7.5+1.936)=0.106$. The other two terms for the numbers taken above are constant and equal to 1.200 and 1.003 respectively. It is clear that the main thermal resistance is offered by the thermal conductivity of zone 2 (weld): the second and third terms on the right-hand side of the equation. The heat transfer from the interior tube surface to the water in the weld zone (the augend in the denominator of the first term) is more intense than the spreading of the heat from the welding zone over the tube wall (the addend in the denominator). This is particularly pronounced with increase in $\alpha$. On the whole, the thermal resistance to heat transfer (the first term of the equation) amounts to about $10 \%$ of the total thermal resistance, which is associated with the high value of the coefficient of heat transfer to boiling water.

From formula (13) it is clear that the overheating of the sheet is affected most strongly by the half-step $S$. If it is doubled $\left(S=81 \cdot 10^{-3} \mathrm{~m}\right)$, the maximum overheating of the sheet $\left(t_{\max }-t_{0}\right)$ will increase four times (from 89 to $345^{\circ} \mathrm{C}$ ) for the lowest heat-flux density $q=35 \mathrm{~kW} / \mathrm{m}^{2}$.

When the step is large, a substantial influence will be exerted by the sheet thickness $\delta_{\text {st }}$ : to decrease the maximum temperature one will have to increase the thickness.

The temperature of the sheet is strongly affected by a decrease in the height $\delta_{\text {weld }}$ and hence the width $F$ of the weld. Thus, with a decrease of 2 mm (from 8 to 6 mm ) in $\delta_{\text {weld }}, F$ decreases from 10 to 7 mm while $t_{\text {max }}$ increases from 345 to $364^{\circ} \mathrm{C}$ for the smallest heat flux and from 747 to $856^{\circ} \mathrm{C}$ for the largest heat flux.

The overheating of the sheet $\left(t_{\max }-t_{0}\right)$ virtually linearly increases with increase in the density of the heat flux $q$ incident on it and with decrease in the thermal-conductivity coefficient $\lambda$.

Of special note is the negative influence of the unwelded gap between the sheet and the lower generatrix of the tube. Were the tube completely welded to the fire sheet, i.e., $\delta_{\text {unw }}=0$ and $F=15 \mathrm{~mm}, t_{\max }$ would vary within $328-656^{\circ} \mathrm{C}$ with the heat-flux density.

But the problem is not only there. Since the steel sheet and the tubes are produced with a certain curvature (allowable according to the All-Union State Standard), the size of the gap between the tube and the sheet will be variable along the tube length. In this case, the weld width average over the tube length should be substituted into formula (13). It may happen that at certain sites there will be no weld at all. If the lengths of such unwelded portions are small (no larger than the step between the tubes) and these portions are distributed more or less uniformly over the tube length, we can evaluate the overheating of the sheet in the first approximation by formula (13), having multiplied $q$ in it by the ratio of the length of the tube to the total length of the portions on which the tubes are welded to the sheet.

If the length of the "weldless" portion is much larger than the step ( $2 S$ ) between the tubes, the temperature precisely at the center of this portion below the tube will be maximum. We can roughly evaluate it by formula (13), having substituted $2 S$ instead of $S$ into it (if the neighboring tubes are normally welded at this site), or we must solve the problem on temperature distribution in a plane sheet with allowance for the real geometry of welding of the tubes.

In the zone of moderate gas temperatures where heat is transferred to the fire sheet mainly by convection, the character of the process corresponds more to the boundary conditions of the third kind $q=k\left(t_{\mathrm{g}}-t\right)$, where $k$ is the coefficient of heat transfer, including the heat transfer from the gases and the thermal resistance of contaminants (since the dust in the gas flow is very sticky, a layer of deposits with a thickness of up to 50 mm is formed on the interior surfaces of the screen; this layer is of critical importance for the value of the heat flux $), k=1 /\left(1 / \alpha_{g}+\delta_{\text {cont }} / \lambda_{\text {cont }}\right)$, and $t$ is the variable-over-the width temperature of the surface of the fire sheet on the source side of the gases.

The temperature of the heating gases $t_{\mathrm{g}}$ is taken to be constant. The coefficient of heat transfer from the tube wall to the boiling liquid will be denoted by $\alpha$ as before.

With the boundary conditions of the third kind, instead of formula (9) we obtain

$$
\begin{equation*}
Q_{\mathrm{st}}=\left(t_{\mathrm{g}}-t_{\mathrm{cent}}\right) \sqrt{k \lambda \delta_{\mathrm{st}}} \tan (m L) \tag{15}
\end{equation*}
$$

where $m=\sqrt{k / \lambda \delta_{\mathrm{st}}}$.
The density of the heat flux approaching the weld (cross section $x=0$ ) is equal, naturally, to

$$
\left.q\right|_{x=0}=k\left(t_{\mathrm{g}}-t_{\mathrm{min}}\right)
$$

and with account for (14) we have

$$
\begin{equation*}
\left.q\right|_{x=0}=k\left(t_{\mathrm{g}}-t_{\text {cent }}\right) /\left(1+0.5 \delta_{\mathrm{st}} / \lambda\right) . \tag{16}
\end{equation*}
$$

Substitution of expressions (2), (15), and (16) into the heat-balance equation (1) yields

$$
\begin{equation*}
\frac{t_{\mathrm{g}}-t_{\mathrm{cent}}}{t_{\mathrm{w}}-t_{0}}=\frac{\alpha F+\sqrt{\alpha \lambda \delta_{\mathrm{t}}}}{\sqrt{k \lambda \delta_{\mathrm{st}}} \tan (m L)+k\left(F+\delta_{\mathrm{unw}}\right) /\left(1+0.5 k \delta_{\mathrm{st}} / \lambda\right)} \tag{17}
\end{equation*}
$$

Solving in the system with Eq. (17) the equation below, which is analogous to (11),

$$
\begin{equation*}
\lambda F\left(t_{\mathrm{cent}}-t_{\mathrm{w}}\right) /\left(0.5 \delta_{\mathrm{st}}+\delta_{\mathrm{weld}}\right)=\left(t_{\mathrm{w}}-t_{0}\right)\left(\alpha F+\sqrt{\alpha \lambda \delta_{\mathrm{t}}}\right) \tag{18}
\end{equation*}
$$

we obtain

$$
\begin{equation*}
t_{\mathrm{g}}-t_{\mathrm{cent}}=\left(t_{\mathrm{g}}-t_{0}\right) B /(1+A), \tag{19}
\end{equation*}
$$

where $A$ and $B$ are dimensionless numbers:

$$
\begin{gathered}
A=\left(\alpha F+\sqrt{\alpha \lambda \delta_{\mathrm{t}}}\right)\left[1 /\left(\sqrt{k \lambda \delta_{\mathrm{st}}} \tan (m L)+k\left(F+\delta_{\mathrm{unw}}\right) /\left(1+0.5 k \delta_{\mathrm{st}} / \lambda\right)\right)+\left(0.5 k \delta_{\mathrm{st}}+\delta_{\mathrm{weld}}\right) / \lambda F\right], \\
B=\left(\alpha F+\sqrt{\alpha \lambda \delta_{\mathrm{t}}}\right) /\left[\sqrt{k \lambda \delta_{\mathrm{st}}} \tan (m L)+k\left(F+\delta_{\mathrm{unw}}\right) /\left(1+0.5 k \delta_{\mathrm{st}} / \lambda\right)\right] .
\end{gathered}
$$

Unlike Eq. (7), the law of change of the temperature along the rod length (portion 1 of the fire sheet) with boundary conditions of the third kind will take the form

$$
\begin{equation*}
\left(t_{\mathrm{g}}-t\right) /\left(t_{\mathrm{g}}-t_{\text {cent }}\right)=\cosh (m(L-x)) / \cosh (m L) \tag{20}
\end{equation*}
$$

Substituting $x=L$ into Eq. (20), we represent the maximum temperature of the sheet at the center of the gap between the tubes as

$$
\begin{equation*}
\left(t_{\mathrm{g}}-t_{\mathrm{max}}\right) /\left(t_{\mathrm{g}}-t_{\text {cent }}\right)=1 / \cosh (m L) \tag{21}
\end{equation*}
$$

Having substituted (19) for the temperature difference $t_{\mathrm{g}}-t_{\text {cent }}$ into relation (21), we will have

$$
\begin{equation*}
\left(t_{\mathrm{g}}-t_{0}\right) /\left(t_{\mathrm{g}}-t_{\max }\right)=(1 / B+A / B) \cosh (m L) \tag{22}
\end{equation*}
$$

or in the final variant we write

$$
\begin{gather*}
\left(\frac{t_{\mathrm{g}}-t_{0}}{\left(t_{\mathrm{g}}-t_{\mathrm{max}}\right) \cosh (m L)}-1\right) \frac{\alpha F}{k \delta_{\mathrm{st}}}=\left(\frac{\text { th }(m L)}{\sqrt{\mathrm{Bi}_{\mathrm{st}}}}+\frac{1}{\left(1+0.5 \mathrm{Bi}_{\mathrm{st}}\right) \delta_{\mathrm{st}} /\left(F+\delta_{\mathrm{unw}}\right)}\right) \times \\
\times\left(\frac{1}{\left(1+\sqrt{\delta_{\mathrm{t}} /\left(\mathrm{Bi}_{\mathrm{b}} F\right)}\right.}+\left(0.5 \delta_{\mathrm{st}}+\delta_{\text {weld }}\right) \mathrm{Bi}_{\mathrm{b}} / F\right) \tag{23}
\end{gather*}
$$

where $\mathrm{Bi}_{\mathrm{b}}=\alpha F / \lambda$ and $\mathrm{Bi}_{\mathrm{st}}=k \delta_{\mathrm{st}} / \lambda$.
For comparison with the calculation performed earlier, we take $k=30 \mathrm{~W} /\left(\mathrm{m}^{2} \cdot \mathrm{~K}\right), \alpha=9000 \mathrm{~W} /\left(\mathrm{m}^{2} \cdot \mathrm{~K}\right)$, and $L$ $=S-F-\delta_{\text {unw }}=40.5-10-5=25.5 \mathrm{~mm}$; the remaining data are the same as in the previous calculation. As a result, we have $\left(t_{\mathrm{g}}-t_{\max }\right)=295^{\circ} \mathrm{C}$.

## CONCLUSIONS

1. Approximate formulas for calculating the temperature field in a fire sheet with welded cooling tubes have been obtained.
2. A detailed analysis of the strongest factors of influence on the value of the overheating of the sheet has been made.
3. An example of calculation of the temperature field as applied to the actual operating conditions of a cop-per-smelting furnace has been given.

## NOTATION

$a_{\mathrm{f} 1}$, emissivity factor; $c_{\mathrm{ash}}$, concentration of ash in the gases, $\mathrm{kg} / \mathrm{m}^{3} ; d_{\mathrm{t}}$, outside diameter of the tube, $\mathrm{m} ; F$, width of the weld, $\mathrm{m} ; f$, cross-sectional area of the tube wall, $\mathrm{m}^{2} ; k$, heat-transfer coefficient, $\mathrm{W} /\left(\mathrm{m}^{2} \cdot \mathrm{~K}\right)$; $k_{\mathrm{b} . \text { att }}$, beam attenuation factor, $1 /(\mathrm{m} \cdot \mathrm{MPa}) ; L$, distance from the center of the gap between the tubes to the weld, $\mathrm{m} ; p$, saturation pressure, Pa; $Q_{\mathrm{st}}$, heat flux arriving along the steel sheet, $\mathrm{W} ; Q_{\mathrm{t}}$, heat flux transferred over the tube wall, $\mathrm{W} ; q$, heatflux density, $\mathrm{W} / \mathrm{m}^{2} ; q_{\mathrm{t}}$, density of the heat flux transferred over the tube wall, $\mathrm{W} / \mathrm{m}^{2} ; S$, half-step between the tubes,
m ; $t$, temperature, ${ }^{\mathrm{o}} \mathrm{C} ; t_{0}$, saturation temperature of the liquid, ${ }^{\circ} \mathrm{C} ; t_{\mathrm{g}}$, temperature of the gases, ${ }^{\mathrm{o}} \mathrm{C} ; t_{\mathrm{max}}$ and $t_{\mathrm{min}}$, maximum and minimum temperatures of the fire-sheet surface, ${ }^{\circ} \mathrm{C} ; t_{\mathrm{w}}$, temperature of the interior wall of the tube in the weld zone, ${ }^{\circ} \mathrm{C} ; t_{\text {cent }}$, temperature at the center of the sheet thickness, ${ }^{\circ} \mathrm{C} ; u$, wetted perimeter, $\mathrm{m} ; w_{\mathrm{g}}$, gas velocity, $\mathrm{m} / \mathrm{sec} ; x$, running coordinate, $\mathrm{m} ; \mathrm{Bi}=\alpha l_{0} / \lambda$, Biot number; $\alpha$, average coefficient of heat transfer from the interior tube surface to the liquid, $\mathrm{W} /\left(\mathrm{m}^{2} \cdot \mathrm{~K}\right) ; \alpha_{\mathrm{g}}$, average coefficient of heat transfer from the gases to the steel sheet, $\mathrm{W} /\left(\mathrm{m}^{2} \cdot \mathrm{~K}\right) ; \Delta$, minimum height of the air gap between the tube and the fire sheet, $\mathrm{m} ; \delta_{\text {cont }}$, thickness of the layer of contaminants, $\mathrm{m} ; \delta_{\mathrm{unw}}$, width of the unwelded gap, $\mathrm{m} ; \delta_{\text {weld }}$, height of the weld, $\mathrm{m} ; \delta_{\mathrm{st}}$, thickness of the steel sheet, $\mathrm{m} ; \delta_{\mathrm{t}}$, thickness of the tube wall, $\mathrm{m} ; \lambda$, thermal-conductivity coefficient of the metal of the tubes, the fire sheet, and the weld, $\mathrm{W} /(\mathrm{m} \cdot \mathrm{K})$; $\lambda_{\mathrm{g}}$, thermal-conductivity coefficient of the gases, $\mathrm{W} /(\mathrm{m} \cdot \mathrm{K})$; $\lambda_{\text {cont }}$, thermal-conductivity coefficient of the contaminants, $\mathrm{W} /(\mathrm{m} \cdot \mathrm{K}) ; \mathrm{v}_{\mathrm{g}}$, coefficient of kinematic viscosity of the gases, $\mathrm{m}^{2} / \mathrm{sec} ; \vartheta$, excess temperature, ${ }^{\circ} \mathrm{C} ; \vartheta_{1}$, excess temperature of the interior wall of the tube in the weld zone, ${ }^{\circ} \mathrm{C}$. Subscripts: g, gases; ash, ash; cont, contaminant; b, boiling; max, maximum; min, minimum; unw, unwelded; b.att, beam attenuation; weld, welding; w, wall; cent, central; st, steel; t, tube; fl, flame; 0, saturation state.

## REFERENCES

1. Thermal Calculation of Boilers (Standard Method) [in Russian], St. Petersburg (1998).
2. V. V. Dobrynin and M. I. Grigor'eva, Prom. Energet., No. 6, 4-5 (2001).
3. V. P. Isachenko, V. A. Osipova, and A. S. Sukomel, Heat Transfer [in Russian], Moscow (1981).
4. V. N. Lukanin, M. G. Shatrov, G. M. Kamfer, et al., Thermal Engineering [in Russian], Moscow (2000).
5. V. N. Yurenev and P. D. Lebedev (eds.), Handbook on Thermal Engineering [in Russian], Vol. 2, Moscow (1976).
